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Spontaneous Lorentz Violation and the Long-Range Gravitational Preferred-Frame Effect

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Abstract

Lorentz-violating operators involving Standard Model fields are tightly constrained by experimental data. However, bounds are more model-independent for Lorentz violation appearing in purely gravitational couplings. The spontaneous breaking of Lorentz invariance by the vacuum expectation value of a vector field selects a universal rest frame. This affects the propagation of the graviton, leading to a modification of Newton's law of gravity. We compute the size of the long-range preferred-frame effect in terms of the coefficients of the two-derivative operators in the low-energy effective theory that involves only the graviton and the Goldstone bosons.

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The possibility of breaking Lorentz invariance is of interest to theoretical physicists for a variety of reasons. For instance, spontaneous Lorentz violation has been proposed at various times as a starting point in alternative theories of electrodynamics [1] and of linear gravity [2], and as a possible solution to the horizon problem in cosmology [3]. Lorentz violation has also been discussed as a potentially observable signal of physics beyond the Planck scale (whether in the context of string theory, noncommutative geometry, or loop quantum gravity) [4], and some researchers have claimed that there is evidence of such violation in the measurements of the energies of cosmic rays [5].

Experimental data puts very tight constraints on Lorentz violating operators that involve Standard Model particles [6], but the bounds are more model-independent on Lorentz violation that appears only in couplings to gravity [7, 8]. One broad class of Lorentz-breaking gravitational theories are the so-called vector-tensor theories in which the space-time metric $g^{\mu\nu}$ is coupled to a vector field S^μ which does not vanish in the vacuum. Consideration of such theories dates back to [9] and their potentially observable consequences are extensively discussed in [10]. These theories have an unconstrained vector-field coupled to gravity. Theories with a unit constraint on the vector field were proposed as a means of alleviating the difficulties that plagued the original unconstrained theories [11].

The phenomenology of these theories with the unit constraint has has been recently explored. It has been proposed as a toy model for modifying dispersion relations at high energy [12]. The spectrum of long-wavelength excitations is discussed in [13], where it was found that all polarizations have a relativistic dispersion relation, but travel with different velocities. Applications of these theories to cosmology have been considered in [14, 15]. Constraints on these theories are weak, as for instance, there are no corrections to the Post-Newtonian parameters γ and β [16]. The status of this class of theories, also known as 'Aether-theories', is reviewed in [17].

Here we begin by considering the general low-energy effective action for a theory in which Lorentz invariance is spontaneously broken by the vacuum expectation value (vev) of a Lorentz four-vector S^μ. With an appropriate rescaling, the vev satisfies

$$\langle S_\mu S^\mu \rangle = 1 \ , \tag{1}$$

since we assume the vev of S^μ is time-like. The existence of this vev implies that there exists a universal rest frame (which we sometimes refer to as the 'preferred frame') in which

$S^\mu = \delta^\mu_0$. When the resulting low-energy effective action is minimally coupled to gravity, we shall see that it simply becomes the vector-tensor theory with the unit constraint.

Objects of mass M_1 and M_2 in a system moving relative to the preferred-frame can experience a modification to Newton's law of gravity of the form [10, 18]

$$U_{\rm Newton} = -G_{\rm N}\frac{M_1 M_2}{r}\left(1 - \frac{\alpha_2}{2}\frac{(\vec{w}\cdot\vec{r})^2}{r^2}\right) \tag{2}$$

where $\vec{w}$ is the velocity of the system under consideration, such as the solar-system or Milky Way galaxy, relative to the universal rest frame. The main purpose of this note is to compute α_2 in theories where Lorentz invariance is spontaneously broken by the vev of a four-vector. This PPN coefficient is more strongly constrained by experiment than the other PPN parameters γ and β [18], so it is natural to focus on it.

The vev of S^μ spontaneously breaks Lorentz invariance. But as rotational invariance is preserved in the preferred frame, only the three boost generators of the Lorentz symmetry are spontaneously broken. The low-energy fluctuations $S^\mu(x)$ which preserve Eq. (1) are the Goldstone bosons of this breaking, i.e., those that satisfy

$$S_\mu(x)S^\mu(x) = 1 \ . \tag{3}$$

In the preferred-frame the fluctuations can be parameterized as a local Lorentz transformation

$$S^\mu(x) = \Lambda^\mu_0(x) = \frac{1}{\sqrt{1-\vec{\phi}^2}}\begin{pmatrix} 1 \\ \vec{\phi} \end{pmatrix} \ . \tag{4}$$

Under Lorentz transformations $S^\mu(x) \to \Lambda^\mu_\nu S^\nu(x)$ and the symmetry is realized non-linearly on the fields ϕ^i. Using this field $S^\mu(x)$ we may then couple the Goldstone bosons to Standard Model fields. Since however, the constraints on Lorentz-violating operators [1] involving Standard Model fields are considerable [6], we instead focus on their couplings to gravity, which are more model independent because they are always present once the Goldstone bosons are made dynamical.

The Goldstone bosons are made dynamical by adding in kinetic terms for them. Since Lorentz invariance is only broken spontaneously, the action for the kinetic terms should

[1] More correctly, operators that appear to be Lorentz violating when the Goldstone bosons ϕ^i are set to zero.

still be invariant under Lorentz transformations. The only interactions relevant at the two derivative-level and not eliminated by the constraint Eq. (3) are

$$\mathcal{L} = c_1 \partial_\alpha S^\beta \partial^\alpha S_\beta + (c_2 + c_3) \partial_\mu S^\mu \partial_\nu S^\nu + c_4 S^\mu \partial_\mu S^\alpha S^\nu \partial_\nu S_\alpha \ . \tag{5}$$

Expanding this action to quadratic order in ϕ^i, one finds that the four parameters c_i can be chosen to avoid the appearance of any ghosts. In particular, we require $c_1 + c_4 < 0$.

With gravity present the situation is more subtle. One expects the gravitons to 'eat' the Goldstone bosons, producing a more complicated spectrum [19, 20]. The covariant generalization of the constraint equation becomes

$$g_{\mu\nu}(x) S^\mu(x) S^\nu(x) = 1 \tag{6}$$

and in the action for S^μ we replace $\partial_\mu \to \nabla_\mu$. Note that there is no "Higgs mechanism" to give the graviton a mass, since the connection is linear in derivatives of the metric.

Local diffeomorphisms can now be used to gauge away the Goldstone bosons. For under a local diffeomorphism (which preserves the constraint Eq. (6)),

$$S'^\mu(x') = \frac{\partial x'^\mu}{\partial x^\nu} S^\nu(x) \tag{7}$$

and with $x'^\mu = x^\mu + \epsilon^\mu$, $S^\mu \equiv v^\mu + \phi^\mu$,

$$\phi'^\mu(x') = \phi^\mu(x) + v^\rho \partial_\rho \epsilon^\mu \tag{8}$$

from which we can determine ϵ^μ to completely remove ϕ^μ. Note that in the preferred frame, ϵ^i can be used to remove ϕ^i. In this gauge, the constraint Eq. (6) reduces to

$$S^0(x) = (1 - h_{00}(x)/2) \ . \tag{9}$$

The residual gauge invariance left in ϵ^0 can be used to remove h_{00}. This is an inconvenient choice when the sources are static. In a more general frame with $\langle S^\mu \rangle = v^\mu$, obtained by a uniform Lorentz-boost from the preferred-frame, the constraint Eq. (6) is solved by

$$S^\mu(x) = v^\mu \left(1 - v^\rho v^\sigma h_{\rho\sigma}(x)/2\right) \ . \tag{10}$$

Next we discuss a toy model that provides an example of a more complete theory, that at low-energies reduces to the theory described above with the vector field satisfying a unit

covariant constraint (6).[2] Consider the following non-gauge invariant theory for a vector boson A^μ,

$$\mathcal{L} = -\frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \nabla_\rho A^\mu \nabla_\sigma A^\nu + \lambda \left(g_{\mu\nu} A^\mu A^\nu - v^2 \right)^2 \ . \tag{11}$$

Fluctuations about the minimum are given by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad , \quad A^\mu = v^\mu + \psi^\mu \ . \tag{12}$$

This theory has one massive state Φ with mass $M_\Phi \propto \lambda^{1/2} v$, which is

$$\Phi = v^\mu \psi_\mu + h_{\mu\nu} v^\mu v^\nu / 2 \ . \tag{13}$$

In the limit that $\lambda \to \infty$ this state decouples from the remaining massless states. In the preferred frame the only massless states are $h_{\mu\nu}$, and ψ^i. Since we have decoupled the heavy state, we should expand

$$A^0 = v + \left[\psi^0 + v h_{00}/2 \right] - v h_{00}/2 \to v - v h_{00}/2 \tag{14}$$

where in the last limit we have decoupled the heavy state. Note that this parameterization of A^0 is precisely the same parameterization that we had above for S^0. In other words, in the limit that we decouple the only heavy state in this model, the field A^μ satisfies $g_{\mu\nu} A^\mu A^\nu = v^2$, which is the same as the constraint (6) with $A^\mu \to v S^\mu$.

In the unitary gauge with $\phi^i = 0$, the only massless degrees of freedom are the gravitons. There are the two helicity modes which in the Lorentz-invariant limit correspond to the two spin-2 gravitons, along with three more helicities that are the Goldstone bosons, for a total of five. The sixth would-be helicity mode is gauged away by the remaining residual gauge invariance.

But the model that we started from does have a ghost, since we wrote a kinetic term for A^μ that does not correspond to the conventional Maxwell kinetic action. The ghost in the theory is A^0, which in our case is massive. The presence of this ghost means that this field theory model is not a good high-energy completion for the low-energy theory involving only S^μ and gravity which we are considering in this letter. We assume that a sensible high energy completion exists for generic values of the c_i's.

[2] For a related example, see [20].

Now we proceed to compute the preferred-frame coefficient α_2 appearing in the modification to Newton's law.

The action we consider is

$$S = \int d^4x \sqrt{g} \left(\mathcal{L}_{\rm EH} + \mathcal{L}_{\rm V} + \mathcal{L}_{\rm gf}\right) \tag{15}$$

with[3]

$$\mathcal{L}_{\rm EH} = -\frac{1}{16\pi G} R \tag{16}$$

$$\mathcal{L}_V \;=\; c_1 \nabla_\alpha S^\beta \nabla^\alpha S_\beta + c_2 \nabla_\mu S^\mu \nabla_\nu S^\nu + c_3 \nabla_\mu S^\nu \nabla_\nu S^\mu + c_4 S^\mu \nabla_\mu S^\alpha S^\nu \nabla_\nu S_\alpha \ , \tag{17}$$

and we use the metric signature $(+---)$. This is the most general action involving two derivatives acting on S^μ that contributes to the two-point function. Note that a coefficient c_3 appears, since in curved spacetime covariant derivatives do not commute. Other terms involving two derivatives acting on S^μ may be added to the action, but they are either equivalent to a combination of the operators already present (such as adding $R_{\mu\nu} S^\mu S^\nu$), or they vanish because of the constraint Eq. (6). We assume generic values for the coefficients c_i that in the low energy effective theory give no ghosts or gradient instabilities.

As previously discussed, S^μ satisfies the constraint (6). We also assume that it does not directly couple to Standard Model fields. In the literature, Eq. (6) is enforced by introducing a Lagrange-multiplier into the action. Here we enforce the constraint by directly solving for S^μ, as given by Eq. (10), and then insert that solution back into the action to obtain an effective action for the metric.

In our approach there is a residual gauge invariance which in the preferred-frame corresponds to reparameterizations involving ϵ^0 only. To completely fix the gauge we add the gauge-fixing term

$$\mathcal{L}_{\rm gf} = -\frac{\alpha}{2} \left(S^\rho S^\sigma S^\mu \partial_\mu h_{\rho\sigma}\right)^2 \ . \tag{18}$$

Neglecting interaction terms, in the preferred frame the gauge-fixing term reduces to

$$\mathcal{L}_{\rm gf} = -\frac{\alpha}{2} \left(\partial_0 h_{00}\right)^2 \ . \tag{19}$$

[3] The coefficients c_i appearing here are related to those appearing in, for example [13], by $c_i^{\rm here} = -c_i^{\rm there}/16\pi G$.

Physically, this corresponds in the $\alpha \to \infty$ limit to removing all time-dependence in h_{00}, without removing the static part which is the gravitational potential. This is a convenient gauge in which to compute when the sources are static.

At the two-derivative level, the only effect in this gauge of the new operators is to modify the kinetic terms for the graviton. The dispersion relation for the five helicities will be of the relativistic form $E = \beta|\vec{k}|$, but where the velocities β are not the same for all helicities and depend on the parameters c_i [13]. This spectrum is different than that which is found in the 'ghost condensate' theory, where in addition to the two massless graviton helicities, there exists a massless scalar degree-of-freedom with a non-relativistic dispersion relation [21].

There exists a range for the c_i's in which the theory has no ghosts and no gradient instabilities [13]. In particular, for small c_i's, no gradient instabilities appear if

$$\frac{c_1 + c_2 + c_3}{c_1 + c_4} > 0 \quad \text{and} \quad \frac{c_1}{c_1 + c_4} > 0 \ . \tag{20}$$

The condition for having no ghosts is simply $c_1 + c_4 < 0$.

The correction to Newton's law in Eq. (2) is linear order in the source. Thus to determine its size we only need to find the graviton propagator, since the non-linearity of gravity contributes at higher order in the source. In order to compute that term we have to specify a coordinate system, of which there are two natural choices. In the universal rest frame the sources, such as the solar system or Milky Way galaxy, will be moving and the computation is involved. We instead choose to compute in the rest frame of the source, which is moving at a speed $|\vec{w}| \ll 1$ relative to the universal rest frame. Observers in that frame will observe the Lorentz breaking vev $v^\mu \simeq (1, -\vec{w})$. In the rest frame of the source, a modified gravitational potential will be generated. Technically this is because terms in the graviton propagator $v \cdot k \simeq \vec{w} \cdot \vec{k}$ are non-vanishing. It is natural to assume that dynamical effects align the universal rest frame where $v^\mu = \delta^\mu_0$ with the rest frame of the cosmic microwave background.

In a general coordinate system moving at a constant speed with respect to the universal frame the Lorentz-breaking vev will be a general time-like vector v^μ. Thus we need to determine the graviton propagator for a general time-like constant v^μ. Since Lorentz invariance is spontaneously broken, the numerator of the graviton propagator is the most general tensor constructed out of the vectors v^μ, k^ν and the tensor $\eta^{\rho\sigma}$. There are 14 such tensors. Writing

the action for the gravitons as

$$S = \frac{1}{2}\int d^4k\, \tilde{h}^{\alpha\beta}(-k)K_{\alpha\beta|\sigma\rho}(k)\tilde{h}^{\sigma\rho}(k) \tag{21}$$

it is a straightforward exercise to determine the graviton propagator $\mathcal{P}$ by solving

$$K_{\alpha\beta|\mu\nu}(k)\mathcal{P}^{\mu\nu|\rho\sigma}(k) = \frac{1}{2}\left(\eta^{\rho}_{\alpha}\eta^{\sigma}_{\beta} + \eta^{\sigma}_{\alpha}\eta^{\rho}_{\beta}\right)\ . \tag{22}$$

The above set of conditions leads to 21 linear equations which determine the 14 coefficients of the graviton propagator in terms of the coefficients c_i and the vev v^μ. Seven equations are redundant and provide a non-trivial consistency check on our calculation.

Although it is necessary to compute all 14 coefficients in order to invert the propagator, here we present only those which modify Newton's law as described previously (assuming stress-tensors are conserved for sources). These are

$$\begin{aligned}\mathcal{P}^{\alpha\beta|\rho\sigma}_{\text{Newton}} \;=\; & \Big\{\mathrm{A}\eta^{\alpha\beta}\eta^{\rho\sigma} + \mathrm{B}(\eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}) + \mathrm{C}(v^\alpha v^\beta\eta^{\rho\sigma} + v^\rho v^\sigma\eta^{\alpha\beta}) \\ & + \mathrm{D}v^\alpha v^\beta v^\rho v^\sigma + \mathrm{E}(v^\alpha v^\rho\eta^{\beta\sigma} + v^\alpha v^\sigma\eta^{\beta\rho} + v^\beta v^\rho\eta^{\alpha\sigma} + v^\beta v^\sigma\eta^{\alpha\rho})\Big\}\end{aligned} \tag{23}$$

We find that each of these coefficients is independent of the gauge parameter α. We also numerically checked that without the presence of the gauge-fixing term the propagator could not be inverted.

To compute the preferred-frame effect coefficient α_2, we only need to focus on terms in the momentum-space propagator proportional to $(v\cdot k)^2$. To leading non-trivial order in $G(v\cdot k)^2$ and in the c_i's we obtain, from the linear combination $A + 2B + 2C + D + 4E$,

$$\begin{aligned} g_{00} \;=\; & 1 + 8\pi G_{\mathrm{N}}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\Big\{1 - 8\pi G_{\mathrm{N}}\frac{(v\cdot k)^2}{k^2}\frac{1}{c_1(c_1+c_2+c_3)}\Big[2c_1^3 + 4c_3^2(c_2+c_3) + \\ & + c_1^2(3c_2+5c_3+3c_4) + c_1((6c_3-c_4)(c_3+c_4) + c_2(6c_3+c_4))\Big]\Big\}\tilde{T}^{00}(k)\end{aligned} \tag{24}$$

where in the first line k is a four-vector. Next we use $v^\mu = (1, -\vec{w})$, place the source at the origin, substitute $T^{00} = M\delta^{(3)}(\vec{x})$ or $\tilde{T}^{00}(k) = 2\pi M\delta(k^0)$ and use

$$\int \frac{d^3k}{(2\pi)^3}\frac{k_ik_j}{\vec{k}^4}e^{i\vec{k}\cdot\vec{x}} = \frac{1}{8\pi r}\left[\delta_{ij} - \frac{x_ix_j}{r^2}\right] \tag{25}$$

to obtain

$$\begin{aligned} g_{00} \;=\; & 1 - 2G_{\mathrm{N}}\frac{M}{r}\Big(1 - \frac{(\vec{w}\cdot\vec{r})^2}{r^2}\frac{8\pi G_{\mathrm{N}}}{2c_1(c_1+c_2+c_3)}\Big[2c_1^3 + 4c_3^2(c_2+c_3) + \\ & + c_1^2(3c_2+5c_3+3c_4) + c_1((6c_3-c_4)(c_3+c_4) + c_2(6c_3+c_4))\Big]\Big)\end{aligned} \tag{26}$$

where we have only written those terms that give a correction to Newton's law proportional to $[\vec{w} \cdot \vec{r}/r]^2$. We have also assumed that $|\vec{w}| \ll 1$ so that higher powers in $\vec{w} \cdot \vec{r}/r$ can be neglected. The factor of $1/c_1$ in the preferred-frame correction to the metric arises because when $c_1 \to 0$ the "transverse" components of ϕ^i have no spatial gradient kinetic term. Similarly, the factor of $1/(c_1 + c_2 + c_3)$ arises because when $c_1 + c_2 + c_3 \to 0$ the "longitudinal" component of ϕ^i has no spatial gradient kinetic term. Either of these cases causes a divergence in the static limit.[4]

The coefficients c_i redefine Newton's constant measured in solar system experiments and we find that

$$G_{\mathrm{N}} = G\left[1 - 8\pi G(c_1 + c_4)\right] \simeq \frac{G}{1 + 8\pi G(c_1 + c_4)} \tag{27}$$

which agrees with previous computations to linear order in the c_i's after correcting for the differences in notation [14, 17].

The experimental bounds on deviations from Einstein gravity in the presence of a source are usually expressed as constraints on the metric perturbation. Since the metric is not gauge-invariant, these bounds are meaningful only once a gauge is specified. In the literature, the bounds are typically quoted in harmonic gauge. Here, the preferred-frame effect is a particular term appearing in the solution for h_{00}. For static sources, the gauge transformation needed to translate the solution in our gauge to the harmonic gauge is itself static. But since a static gauge transformation cannot change h_{00}, we may read off the coefficient of the preferred-frame effect in the gauge that we used.

By inspection

$$\begin{aligned} \alpha_2 \ = \ & \frac{8\pi G_{\mathrm{N}}}{c_1(c_1 + c_2 + c_3)} \Big[2c_1^3 + 4c_3^2(c_2 + c_3) + c_1^2(3c_2 + 5c_3 + 3c_4) \\ & + c_1((6c_3 - c_4)(c_3 + c_4) + c_2(6c_3 + c_4))\Big] \ , \end{aligned} \tag{28}$$

which can be compared with the experimental bound $|\alpha_2| < 4 \times 10^{-7}$ given in [18].

A considerably stronger constraint on the size of the c_i's can probably be obtained from the gravitational Cherenkov radiation of the highest-energy cosmic rays [7].

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[4] This divergence can of course be avoided by considering higher-derivative terms in the action for the Goldstone bosons. This would then give non-relativistic dispersion relations for these modes (as was the case in [21]).

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